Electric Potential

Physics Topics
If necessary, review the following topics and relevant textbook sections from Serway / Jewett “Physics for Scientists and Engineers”, 9th Ed.

- Electric field from plane of charge (Serway Example 24.5)
- Electric Potential (Serway 25.1 - 25.4)

Introduction

Review of Fields Concept
We have seen that for gravity and electromagnetism, it is helpful to understand the field as a way to quantify how a masses or charges exert forces on each other without touching. Masses/charges feel a force due to the gravitational/electric field. The field is the force on a mass $m$ (or a charge $q$) divided by that mass (or charge),

$$\vec{g} = \frac{\vec{F}_g}{m}$$
$$\vec{E} = \frac{\vec{F}_E}{q}$$

Another way of saying this is that the gravitational field is the “gravitational force per mass”. The electric field is the “electric force per charge”.

Introduction To Electric Potential

There is another quantity called electric potential which is also defined on a “per charge” basis. Electric potential is defined as “potential energy per charge”. In physics, only changes in potential energy matter so we define the potential difference, or voltage difference between two points as the change in potential energy of a charge moved between those two points divided by the value of its charge

$$\Delta V = \Delta U/q$$

Note, the units of electric potential are energy per charge or Joules/Coulomb. The name for this unit is the Volt. (1V = 1 J/C).
A Gravitational Analogy

The closest analogy to potential for gravity is “height”. Lifting up a 1kg mass by 1.0m will increase its potential energy by 9.8J. The change in potential energy per mass is $9.8\text{J}/1\text{kg} = 9.8\text{J/kg}$. Lifting up a 2kg mass by 1.0m will increase its potential energy by $2 \times 9.8\text{J} = 19.6\text{J}$. The change in potential energy per kg is $19.6\text{J}/(2\text{ kg}) = 9.8\text{ J/kg}$. Note that even though the heavier mass has a higher potential energy than the lighter mass, because they are at the same height they have the same gravitational potential. We often represent heights on 2D maps with lines of constant elevation (a topographic map).

![Figure 1 - A Topographic Map.](https://www.greenbelly.co/pages/contour-lines)

We can do the same for electricity. Moving a charge $+2\text{C}$ from point A (low potential) to point B (high potential) will require more energy than moving a charge $+1\text{C}$ from point A to point B. Thus, the larger charge will have a bigger change in potential energy than the smaller charge, but they will both have the same change in electric potential since they start and end at the same points. When drawing a map for electricity, in this case, the lines are not constant height, but lines of constant voltage (electric potential). We call these lines “equipotential lines”.

In this lab, you will use metal electrodes (which act like “charges” when they are hooked to a battery) and conductive paper to try to map the electric potential between two “point charges” and between two metal bars which act like parallel plates. You will also investigate the potential difference as you move along the center line between the charges.

Point charges

In 3-dimensions, the electric potential $V$ at a distance $r$ from a point charge $Q$ is given by

$$V(r) = k_e Q/r$$

(4)

If there are multiple charges nearby, the total potential a point $P$ is found by summing the individual potentials $V_{\text{tot}} = k_e Q_1/r_1 + k_e Q_2/r_2$ where $r_1$ is the distance between $P$ and $Q_1$ and $r_1$
is the distance between $P$ and $r_2$. However, in this lab, we will be using conductive paper which is a 2-dimensional surface. On such a surface the electric potential at a point $P$ away from a “point” charge is given by

$$V(r) = 2KQ \ln(r_0/r) \quad (5)$$

where $K$ is a constant, and $r_0$ is also a constant reference point (note that the potential at position $r_0 = 0$). To find the total potential, you can still add up the potential from two point charges $V_{\text{tot}} = V_1 + V_2 + \ldots$.

“Infinite” Plates

In three dimensions, the field between two large plates of charge is approximately constant. When confined to a 2-dimensional plane, the field between two long lines of charge will also be approximately constant. In such situations, the potential difference between the bar at lower potential and a point $P$ a distance $\Delta x$ away from it is

$$\Delta V = |\vec{E}| \Delta x \quad (6)$$

where $|E|$ is the magnitude of the electric field in between the bars/plates.

Pre-Lab Questions

Please complete the following questions prior to coming to lab. At the beginning of lab, you will be given a short quiz which is heavily based on one (or more) of these questions.

1.) Suppose you are in the vicinity of two point charges in 3 dimensions $+Q$ and $-Q$. The negative charge has $(x, y)$ coordinates $(0, 0)$ while the $+Q$ charge has coordinates $(0, d)$. Using equation (4), write the total electric potential at a point in between the charges (along a line connecting them), a distance $r$ away from the negative charge.

2.) Suppose you are in the vicinity of two point charges $+Q$ and $-Q$ in 2 dimensions. The negative charge has $(x, y)$ coordinates $(0, 0)$ while the $+Q$ charge has coordinates $(0, d)$. Using equation (5), write the total electric potential at a point in between the charges (along a line connecting them), a distance $r$ away from the negative charge.

3.) Simplify your result from the previous question using the rules for logs $\ln(AB) = \ln A + \ln B$ and $\ln(A/B) = \ln(A) - \ln(B)$.

4.) If you took data of $V_{\text{total}}$ and $r$, how could you plot your data so as to form a straight line? What would be the slope of this line?

5.) Suppose you moved a charge of $+1C$ from point A (on a 10V equipotential line) to point B (also on a 10V equipotential line).

(a) How much would its electric potential energy change? Explain.
(b) How much would its electric potential change? Explain.
(c) How (if at all) would the answers to the previous two questions change if the charge had magnitude 2C? Explain

6.) Suppose you moved a charge of +3C from point A on a 10V equipotential line to point B on a 20V equipotential line.

(a) How much would its electric potential change? Give a value and explain whether it increased or decreased.
(b) How much would its electric potential energy change? Give a value and explain whether it increased or decreased.
(c) How (if at all) would the answers to the previous two questions change if the charge had magnitude 2C? Explain.

Apparatus
- PASCO Conductive paper
- Two D-cell batteries in series (or power supply)
- Metal electrodes (with embedded magnets to ensure good contact)
- Magnetic board on which paper will be placed.
- Banana cables and alligator clips
- Vernier Logger Pro software
- Vernier Computer interface (LabQuest, LabQuest mini, etc.)
- Vernier Differential Voltage Probe

Procedure
Preliminary Setup and Exploration
1.) Connect the differential voltage probe to CH-1 of the Vernier computer interface.
2.) Open the LoggerPro software (no specific file is necessary). You should see a box in the lower right hand corner of the screen reading the voltage difference (electric potential difference between the two leads of the differential voltage probe).
3.) Touch the red and black leads of the differential voltage probe together. When the two leads are touched together, they should be at the same electric potential. Since the probe reads the difference between the leads, it should read zero when the leads are touched together. Select Experiment → Zero to zero the probe.
4.) Touch the black lead of the differential voltage to the (−) end of one of the D-cell batteries, and the red lead to the (+) side of one of the D-cell batteries. The probe should read in the range 1.2 – 1.5V for each D-cell battery. If your reading is significantly less than this, contact your lab instructor, you may need replacement batteries.
5.) Switch the leads so the black lead is touching the (+) side of the battery and the red end is touching the (−) side. Record your observations.

Part I - “Point-like” electrodes

Plotting equipotential Lines

1.) Place two of the small circular magnets on the conductive paper and secure in place. Your electrodes should be at least 20 cm apart, lying on the same horizontal line. Record the \((x, y)\) coordinates of each magnet.

2.) Using cables and alligator clips, connect the (+) side of the batteries to the right magnet, and the (−) side of the batteries to the left magnet. The magnets are now acting like point charges. **Important: we are using magnets as a simple way to attach a conductive piece of metal to the conductive paper, but this lab does not have anything to do with magnetic fields. We are measuring electric potential. The lab would still work with non-magnetic metals attached to the paper in a different way.**

3.) Connect the black lead of the probe to the (−) electrode. Touch the red lead to the opposite (+) metal electrode. The voltage reading should be somewhere in the range 2.8 - 3.5V. If this is not the case, make sure all contacts are secure and try again. If your readings are significantly different than this, ask your lab TA for help.

4.) Touch the red lead of the differential voltage probe to the (−) metal electrode (but not directly touching the black lead of the probe). Record your observation.

5.) Download and open the Excel Spreadsheet file PCS125LabPotential.xslx. The file should be available on the physics lab website.

6.) Immediately save the file with a new file name which includes your name and the word “point”, for example: PCS125LabPotentialSmith_Point.xlsx

7.) Keeping the black lead of the probe attached to the (−) electrode, touch the red lead of the probe to a point directly below the (−) electrode. Record the voltage at this point and enter it into the orange box on the spreadsheet. Enter the \((x,y)\) coordinates of the point in green area of the spreadsheet.

8.) By moving to the right and upward, find another point which has the same value of the electric potential as you found in the previous step.

9.) Do your best to estimate the coordinates of the point you found if it does not exactly lie on a “dot” on the conductive paper. Enter the coordinates into the the same column as previously
10.) By systematically moving around the electrode in a counterclockwise fashion, repeat steps (8) - (9), recording the coordinates of at least 5 points which have the same potential as the original point in the Excel spreadsheet. As you enter each of the coordinates into the spreadsheet you should see a line being plotted on a graph. Try to get a good range of points so as to make clear what the shape of the equipotential line is.

11.) Go back a point directly below the \((-\)\) electrode and find a different starting point which differs from your previous voltage by at least 0.2V. Repeat steps (7) - (9) for this next equipotential line.

12.) Continue this process until you have plotted at least 3 equipotential lines. Try to keep the interval between your equipotential lines constant (for example, every 0.2V).

13.) Save your file after you have finished taking data and make sure to copy it to a USB disk or e-mail it to yourself and/or your partner so data does not get lost.

\(V(x)\) along center line for point like electrodes

1.) Keeping the black lead of the probe attached to the \((-\)\) electrode, place the red lead of the probe 1.0 cm to the right, along the line between the two electrodes.

2.) In a spreadsheet and/or in your lab notebooks, record the voltage reading, and the distance \(r\) from the \((-\)\) electrode.

3.) Now increase the distance between the read lead and the \((-\)\) electrode by 1-2 cm. Record the corresponding voltage. Continue taking data until you have a set of data \(r\) vs. Voltage. Stop when you are 1cm away from the \((+)\) electrode.

4.) Save your file after you have finished taking data and make sure to copy it to a USB disk or e-mail it to yourself and/or your partner so data does not get lost.

Part II - “Bar-like” electrodes

Plotting equipotential lines

1.) Place two of the longer metal bars (oriented vertically) on the conductive paper and firmly secure in place. Your electrodes should be at least 20 cm apart, with their centers on the same horizontal line. Keep the bars the same distance apart as you had your “point charges”. Record the \((x, y)\) coordinates of centers of the bars in your notebook.

2.) Using cables, connect the \((+)\) side of the batteries to the right electrode, and the \((-\)\) side of the batteries to the left electrode.
3.) Touch the red lead to the opposite (+) metal electrode. The voltage reading should be between 2.8-3.5V. If this is not the case, make sure all contacts are secure and try again. If your readings are still significantly outside of this range, ask your lab TA for help.

4.) Touch the red lead of the differential voltage probe to the (−) metal electrode (but not directly touching the black lead of the probe). Record your observation.

5.) Download another copy of the Excel Spreadsheet file PCS125LabPotential.xlsx. The file should be available on the physics lab website.

6.) Immediately save the file with a new file name which includes your name, and the word “bars” for example PCS125LabPotentialSmith_Bars.xlsx

7.) Repeat all steps as you did previously for the “point-like” electrodes for these “bar” electrodes. Plot at least 3 equipotential lines. Save your file after you have finished taking data and make sure to copy it to a USB disk or e-mail it to yourself and/or your partner so data does not get lost.

V(x) along center line for bar electrodes

1.) Keeping the black lead of the probe attached to the (−) electrode, place the red lead of the probe 1.0 cm to the right, along the line between the two electrodes.

2.) In a spreadsheet and/or in your lab notebooks, record the voltage reading, and the distance r from the (−) electrode.

3.) Now increase the distance between the read lead and the (−) electrode by 1-2 cm. Record the corresponding voltage. Continue taking data until you have a set of data r vs. Voltage. Stop when you are 1cm away from the (+) electrode.

4.) Save your file after you have finished taking data and make sure to copy it to a USB disk or e-mail it to yourself and/or your partner so data does not get lost.

Analysis

1.) Using MS Excel (or any program you choose), plot a graph of V vs. distance r away from the (−) electrode along the center line connecting the electrodes.

2.) On the same graph, plot the data for the point-like electrodes and the bar electrodes.

3.) Using your answers to the pre-lab questions, on a different graph, plot your “point like” electrode voltage vs. distance graph in such a way as to form a linear graph.

4.) Fit the graph you made in the previous part with a linear line. Assuming K has the same numerical value as Coulomb’s constant in 3D (but different units) (K = 9 × 10^9 N · m/C^2), how much positive charge Q is on the (+) electrode?
5.) Fit the graph of the “bar” electrodes voltage vs. distance with a line. Determine the approximate value of the electric field between the bars.

Wrap Up
The following questions are designed to make sure that you understand the physics implications of the experiment and also to extend your knowledge of the physical concepts covered. Each member of your group should be able to answer any/all of these questions. Your TA will check that this is the case; please check out with your TA before exiting lab.

1.) Compare your voltage vs. distance graphs for your point electrodes and your bar electrodes. Which is more linear? Does this make sense according to equations (5) and (6)?

2.) Equipotential lines cannot cross. Explain why not.

3.) Looking at your equipotential lines for your bar electrodes, what would the lines look like if you used bars which were twice as long? What would the lines look like if the bars were extremely (infinitely) long? Explain.

4.) Working with nearby groups, set up your bar electrodes again and hook them to the batteries as before. Now, using 4 additional bars, create a closed “box” in the middle of the conductive paper. With the black lead of the probe on the (−) electrode measure the electric potential at various points inside the box you have made. Record your observations. Do you have any explanation for what you see?

Report
Follow your TAs instructions on how to submit a report on the work you did. The report is a record of the data/analysis of the experiment, but does not need a lengthy introduction, conclusion, or procedure section. This informal report should include:

- **Identifying Info** - On the official Ryerson lab cover page, include your name, your partner(s) names, the date, course and section, TA name, and the name of the experiment.

- **Objective** - One or two sentence explanation of goal(s) of the experiment and how those goals will be acheived.

- **Data** - Lists and/or data table(s) showing all measurements taken, including units and uncertainties. If you used any techniques not listed in the lab manual or encountered unexpected issues, briefly comment on these in your report. Save any LoggerPro or Excel files and submit them along with your data as a separate file. **Your report should include 4 sets of data with associated graphs: 2 equipotential graphs (for point charges and bars), and 2 sets of data for Vus,r along the line between electrodes (for point charges and bars).**
• **Analysis** - Sample calculations to demonstrate to your TA/instructor how your analysis was completed. If you propagated uncertainty in any calculations, a sample of each type of calculation should be included. Include properly labeled graphs, including error bars to indicate uncertainty (if applicable). If you used any non-standard techniques, explain your methodology in words.

• **Results** - Your main results. If necessary include percent error calculations and/or uncertainty. *Briefly* comment on any unexpected results.

• **Wrap up** - A brief (few sentences) response to each of the wrap-up questions.