Intermediate DC Circuits

Introduction

Without a doubt electronics play an integral part of our lives. Many (if not all) electronics used today (in one way or another) stem from the principles covered in this lab. To begin to understand electronics, we must first understand each electronic component and how these components function with one another. When these components are connected, it is often referred to as a network (or circuit) and the study of these is called network (or circuit) theory. Using circuit theory, one can understand how each component behaves in a network.

Common components (not necessarily used in this lab) include voltage supplies, current supplies, capacitors, inductors, resistors, etc. These components can be separated into two categories: sources, and loads. Sources are components that will output the designated electrical quantity (voltage or current) \textit{ad infinitum} whereas loads will ‘use’ a certain amount of these quantities.

This lab uses Direct Current (DC) sources. Unlike its counterpart Alternating Current (AC) sources, DC sources provide a constant, unchanging output of the designated source. The benefit of studying circuits using DC sources is that the circuit reaches a time-independent steady state.

The primary objective of this lab is to build circuits (of increasing complexity) to determine the resistance of several resistors. First using Ohm’s Law, then by a voltage divider, and finally using a bridge circuit. Lastly, an extended concept is introduced to further extend your knowledge of circuit theory.

Apparatus

- Digital Multimeter (DMM)
- Power Supply
- Decade Resistor (0-10 MΩ± 1%)
- Resistors (3)
- High precision resistor
- Alligator-Banana Cables
- Fractional potentiometer (0-1 kΩ± 5%)
- Banana-Banana Cables
- Breadboard + pins

Decade Resistor - The decade resistor is an array of resistors of differing by order of magnitude ($10^n$ hence decade) allowing one to create resistances of many different values. Next to each knob, there is an order of magnitude and below shows the schematic of each decade connected in series.

Electrical Prototyping Board - Commonly referred to as a breadboard, the electrical prototyping board components can be inserted into the board and connected to other components (without the need for more permanent connecting). The board typically consists of...
two rails that run the length of the board and are typically reserved for power (commonly red) and ground (commonly blue). On each rail, each pin hole is connected to one another. The centre of the board consists of a lattice, separated in half lengthwise by a groove. It is numbered and lettered for pin hole reference. On this part of the lattice, pin holes sharing the same number, on the same side of the groove, are connected within the breadboard.

**Digital Multimeter** - The digital multimeter has a primary knob in the centre that indicates the function, and range (of said function) of the digital multimeter. The bottom of the multimeter has four leads which change depending on which function is selected. The digital multimeter will be used as an ohmmeter, an ammeter, and a voltmeter. The symbols respectively are as follows:

\[ \Omega \quad A \quad V \]

**Part I - Ohm’s Law and Resistors**

The resistor is an electrical component that is used to impede (or resist) the flow of electric current. It is defined by Ohm’s Law stating that the proportionality of voltage \( V \) to current \( I \) by resistive component \( R \).

\[ V = IR \quad (1) \]

Commercial resistors come in several different form factors and markings. The primary ones used in this lab are axial-lead resistors with colour code markings. The colour code is a universally accepted code with each colour relating to a number. Determining the resistor value from the colours can be done by the following:

1.) Determine the number of colour bands on the resistor. It will be either 4 bands, or 5/6 bands.

2.) Determine the correct direction to read the resistor - the larger gap between the bands should be closer to the right.

3.) The left most band indicates the *most* significant digit.

4.) The band to the right indicates the *next most* significant digit. For 5/6 bands, there is an extra significant digit for more precision (hence the extra band). This band as one would guess is the next band to the right.

5.) The last colour band in this cluster (left of the larger gap) is the multiplier. The value indicates the order of magnitude \( 10^n \) of the resistor.

6.) The band(s) to the right of the gap indicate tolerance, and temperature coefficient (6th band if present).
Reading Resistors and using an Ohmmeter

Throughout the lab, we will be measuring the resistance of several resistors using increasingly complicated circuits. Each method will be briefly discussed to see the limitations. We begin by first reading the rated resistance value by colour code, and verifying the resistance by ohmmeter. Although the internal circuit is not discussed in this lab, we can accept that it is a good metric in determining the resistance values in part due to its ease of use.

1.) Create a table with entries for each resistor you measure. Create columns for each method, and each uncertainty associated with each method. Leave room for extra columns (which will be added as you proceed).

2.) Determine the value of the resistors by using the colour band code. Record the tolerance in the uncertainty column for this method.

3.) Determine the resistance value (and uncertainty) for each resistor using the digital multimeter and two banana-alligator cables. Be sure that the DMM knob is set to ohmmeter mode, and the correct terminals are used. **Hint: Use the colour code to select the correct range.**

<table>
<thead>
<tr>
<th>Colour</th>
<th>( n_{1,2,3} )</th>
<th>( 10^p )</th>
<th>Tolerance</th>
<th>( T_c ) [ppm/K]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Black</td>
<td>0</td>
<td>( 10^0 )</td>
<td>±1%</td>
<td>250</td>
</tr>
<tr>
<td>Brown</td>
<td>1</td>
<td>( 10^1 )</td>
<td>±1%</td>
<td>100</td>
</tr>
<tr>
<td>Red</td>
<td>2</td>
<td>( 10^2 )</td>
<td>±2%</td>
<td>50</td>
</tr>
<tr>
<td>Orange</td>
<td>3</td>
<td>( 10^3 )</td>
<td>±1%</td>
<td>25</td>
</tr>
<tr>
<td>Yellow</td>
<td>4</td>
<td>( 10^4 )</td>
<td>±0.5%</td>
<td>20</td>
</tr>
<tr>
<td>Green</td>
<td>5</td>
<td>( 10^5 )</td>
<td>±0.25%</td>
<td>10</td>
</tr>
<tr>
<td>Blue</td>
<td>6</td>
<td>( 10^6 )</td>
<td>±0.1%</td>
<td>5</td>
</tr>
<tr>
<td>Violet</td>
<td>7</td>
<td>( 10^7 )</td>
<td>±1%</td>
<td>1</td>
</tr>
<tr>
<td>Grey</td>
<td>8</td>
<td>( 10^8 )</td>
<td>±0.5%</td>
<td></td>
</tr>
<tr>
<td>White</td>
<td>9</td>
<td>( 10^{-1} )</td>
<td>±5%</td>
<td></td>
</tr>
<tr>
<td>Gold</td>
<td></td>
<td>( 10^{-2} )</td>
<td>±10%</td>
<td></td>
</tr>
<tr>
<td>Silver</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
4.) Discuss the fundamental difference between the tolerance from the colour band method and uncertainty (of measurement).

5.) Comment on whether or not the resistance values are within tolerance.

**Measuring Resistance using Ohm’s Law**

Using Ohm’s Law, one can determine the resistance provided the voltage $V$ and current $I$ are known.

1.) Turn the power supply on and set the voltage to 10 V - remember to turn up the current dial so that it is not completely off. **Don’t forget to note the uncertainty of the voltage**

2.) Switch the digital multimeter knob to ammeter mode, and connect cables to the correct terminals (**Hint: Look at the labelling around the terminals. Hint: Use the colour band resistance, and Ohm’s Law to select the correct range.**

3.) Create the following circuit using a banana-banana cable, and two banana-alligator cables. It’s good practice to have the power supply off when connecting/disconnecting cables to and from it.

4.) Measure and record the current from the digital multimeter.

5.) Using the value of the set voltage, and electric current, calculate the resistance value for each resistor. Calculate the uncertainty using this method as well.

6.) Comment on the ability to determine resistance values using this technique. Where did it seem to work and not work? What can be said about this method in practice that is not realized in theory?
Part II - Kirchhoff’s Laws, and Equivalent Resistors

To analyse and create more complex circuit systems, we must use Kirchhoff’s circuit laws. They are summarized below:

- **Kirchhoff’s Current Law (KCL)** - At a given point (more importantly at nodes or junctions), the sum of currents must equal 0.

\[ \sum_{k=1}^{n} I_n = 0 \]  

(2)

By extension, components in series with one another will have the same amount of current flowing through them.

- **Kirchhoff’s Voltage Law (KVL)** - In any closed (circuit) loop, the sum of voltages must equal 0.

\[ \sum_{k=1}^{n} V_n = 0 \]  

(3)

By extension, components in parallel with one another will have the same voltage across them.

For resistors, the consequence of this law allow for one to create an ‘equivalent’ resistor dependent on how the resistors are configured with one another. The two configurations are **series** and **parallel**. Any other configuration is a combination of these two configurations.

- For resistors in series

\[ R_{eq,s} = \sum_{n} R_n \]  

(4)

- For resistors in parallel

\[ \frac{1}{R_{eq,p}} = \sum_{n} \frac{1}{R_n} \]  

(5)

**Measuring Resistance using Voltage Dividing**

Using Kirchhoff’s laws, we can develop a slightly more advanced circuit to determine the value of a resistor. We develop two different circuits.

1.) Determine the resistor value using two resistors in **series**.

   (a) Using Kirchhoff’s laws, derive an equation for the voltages, \( V_1 \) and \( V_2 \), across two resistors, \( R_1 \) and \( R_2 \), in **series** connected to a voltage supply \( V \).
(b) From your derivation, what can you say about the distribution of voltage across each resistor. **Hint:** Observe the fraction created by the resistor terms.

(c) Using the breadboard, create the circuit drawn above. Use the high precision resistor (5 band code) as $R_1$ and any other resistor used in Part I for $R_2$. Place pins in the breadboard at nodes to allow for easy attachment of external components (such as the power supply, and voltmeter).

(d) Set the power supply to 10 V.

(e) Using the voltage measured across $R_2$, determine the resistance value.

(f) Record the uncertainty using this method.

(g) Replace $R_2$ with a different resistor and repeat the process for all other resistors.

(h) Once again, comment on the ability to determine resistance values using this technique. Where did it seem to work and not work? **Hint:** Consider what variables you have in the equation you used to solve for $R_2$ and what you could change/replace.

2.) Append more columns to the table created in Part I. These columns should be used to record results of the following process.

3.) Determine the resistor value using two resistors in parallel.

   (a) Using Kirchhoff’s laws, derive an equation for the voltages across two resistors $R_1$ and $R_2$ in parallel connected to a voltage supply $V$.

   (b) From your derivation, what can you say about the distribution of current across each resistor. **Hint:** Observe the fraction created by the resistor terms.
(c) Once again, create the circuit above. Use the high precision resistor as $R_1$ and any other resistor used in Part I for $R_2$. Set the power supply to 10 V.

(d) Measure the total current by using the ammeter. This must be done as the power supplies’ indicator for current isn’t sufficiently precise. **Hint:** Identify where total current is present in the circuit and insert the ammeter at that point.

(e) Using the current measured across $R_2$, determine its resistance.

(f) Record the uncertainty using this method.

(g) Comment on the validity of this method in determining the resistance value. **Hint:** Is there a difference between doing this compared to Part I? Is that difference beneficial?

**Part III - The Wheatstone Bridge**

In previous sections, more simple circuits are used to determine the resistance value of resistors. The uncertainty of those methods heavily relied on the precision of the voltage, and current values measured.

A Wheatstone bridge circuit can be used to very accurately determine the resistance of unknown resistors using known resistors. This is achieved by ‘balancing’ the electrical current between two branches of the circuit. Consider the following circuit made from one unknown resistor $R_x$, two known resistors $R_A$, $R_B$, and an adjustable resistor (potentiometer) $R_{pot}$.

![Wheatstone Bridge Circuit Diagram](image)

The resistance of the unknown resistor $R_x$ is determined by considering the nodes $P$ and $Q$, and the two loops on the circuit. By Kirchhoff’s Laws:

\[
0 = I_B - I_x - I_{PQ} \text{ at } P \quad (6)
\]
\[
0 = I_A - I_{pot} + I_{PQ} \text{ at } Q \quad (7)
\]
\[
0 = V_A - V_B - V_{PQ} \text{ top loop} \quad (8)
\]
\[
0 = V_{pot} - V_x + V_{PQ} \text{ bottom loop} \quad (9)
\]
Setting the condition that branch $P$ and branch $Q$ are balanced:

$$
I_{PQ} = 0 \quad (10)
$$
$$
V_{PQ} = 0 \quad (11)
$$

The above equations become (with some rearranging):

$$
I_B = I_x \quad (12)
$$
$$
I_A = I_{pot} \quad (13)
$$
$$
V_A = I_A R_A = I_B R_B = V_B \quad (14)
$$
$$
V_{pot} = I_{pot} R_{pot} = I_x R_x = V_x \quad (15)
$$

Solving for $R_x$:

$$
R_x = \frac{I_{pot} R_{pot}}{I_x} \quad (16)
$$
$$
= \frac{I_A}{I_B} R_{pot} \quad (17)
$$

$$
R_x = \frac{R_B}{R_A} R_{pot} \text{ when } I_{PQ} = 0, V_{PQ} = 0 \quad (18)
$$

The result is the value of $R_x$ is purely dependent on the other resistors. By adjusting the resistor $R_{pot}$, the branches are balanced to meet the conditions. The ratio of $R_B$ and $R_A$ serve as a scaling factor. In theory, this factor can be set to 1 but on application, since there is a finite range for $R_{pot}$, if $R_x$ is outside the range of $R_{pot}$ the branches would never be balanced. In this lab, a decade resistor is used for $R_B$ to permit discrete scaling for $R_{pot}$.

**Measuring Resistance using a Wheatstone Bridge**

Using the Wheatstone Bridge, we determine the resistance values for the resistors used in Part I and II.

1.) On the breadboard, create the top loop using the high precision resistor for $R_A$, the decade resistor for $R_B$, and the DMM for the ammeter. Use pins inserted into the pin holes at each node for easier connecting.

2.) Complete the circuit by completing the bottom loop using the potentiometer for $R_{pot}$, and a resistor you used in Part I for $R_x$. Carefully inspect the side of the potentiometer to determine how it should be connected.

3.) Knowing roughly the value of $R_x$ (colour code), set the ratio $\frac{R_B}{R_A}$ to a value such that $R_x$ can be measured in the range of $R_{pot}$.
4.) Adjust the potentiometer until you obtain the smallest possible current reading on the ammeter. You might want to start on a higher setting for the DMM until you get close to zero and then switch to a smaller range to maximize your precision.

5.) When you are satisfied where the potentiometer is set, determine the resistance value of $R_{pot}$. The potentiometer is designed to take 10 full turns to travel the full range. The dial indicates how many turns (and fraction of a turn) you have travelled.

6.) Calculate the resistance of $R_x$ and add it as an extra column to Part I.

7.) Calculate the uncertainty of $R_x$ using the uncertainties of the other resistors. Refer to the Apparatus section to see the ratings.

8.) Repeat this process for the other resistors.

9.) Comment on the ability to determine resistance values using this technique. What can be said about the precision of this measurement; particularly with respect to the order of magnitude of the resistors tested?

Part IV - Thévenin’s Theorem

In more advance circuits, determining the values (of voltage, and current) within the circuit using Kirchhoff’s Laws can sometimes become quite difficult. This is often from the fact that more complex circuits require solving many linear equations (from using Kirchhoff’s laws). Moreover, if a component in the circuit were to be changed, one would need to recalculate the entire system of equations to determine the solution.

A particular benefit to using Thévenin’s Theorem is when the desire is to determine the electrical properties to a singular component within a circuit. In essence, the rest of the circuit can be expressed by a simplified circuit consisting of a single source, and a load.

The Thévenin equivalent circuit is calculated using the following method:

1.) Replace any components directly parallel to the two points of interest with an open circuit. This is justified by Kirchhoff’s Voltage Law.

2.) Calculate the voltage drop between the two points - this is the Thévenin equivalent voltage.

3.) Substitute all voltage sources with a short circuit, and all current sources with an opening circuit.

4.) Calculate the equivalent resistance of the modified circuit - this is the Thévenin equivalent resistance.

5.) The Thévenin equivalent circuit is then the Thévenin equivalent voltage in series with the Thévenin equivalent resistance connected to the points of interest.

*Or a collection of components (recall equivalent resistors)
If we can verify that the electrical properties between points $A$ and $B$ remain (relatively) unchanged when replacing the original circuit with a Thévenin equivalent circuit, then the theorem is valid.

1.) Build the following circuit using the resistors provided. The circuit must be made correctly but what you choose as each resistor is entirely your choice. Set the voltage supply to 10 V.

2.) Using the voltmeter, measure the voltage between point $A$ and $B$.

3.) Using the ammeter, measure the current flowing between $A$ and $B$.

4.) Calculate the voltage between points $A$ and $B$. Recall that this is the Thévenin Voltage. **Hint: Remember to replace $R_3$ with a short circuit**

5.) Calculate the Thévenin equivalent resistance by replacing the voltage source with a short circuit.

6.) Using the power supply, and the decade resistance box, create the Thévenin circuit by setting the voltage to the Thévenin voltage and resistance to the Thévenin resistance.

7.) Create the Thévenin circuit and once again measure the voltage across $A$ and $B$ and current through $A$ and $B$. 
8.) Was Thévenin’s Theorem verified? Assuming it was, discuss possible sources that could have contributed to the error. **Hint:** We assumed our voltage source was a perfect one hence replacing it with an open circuit.

**Last Few Steps**

1.) Tidy up your work station, turn off the power supply (after setting values to 0), unplug all the components, and ensure the DMM is turned off.