

SIGNIFICANT FIGURES

Every experiment (except in some counting situations) involves a degree of uncertainty. Thus, suppose that several people measure the length of a sheet of paper, using a ruler that is divided into tenths of a centimeter, and get the following results: 27.92 cm, 27.96 cm, 27.90 cm, 2.10 cm. Notice that everyone agrees as to the first three digits (except the last person who is measuring from the wrong end of a 30 cm long ruler!). Clearly the fourth digit, which has been estimated by everyone, is a doubtful figure (in fact even the third figure may be doubtful if the ruler is too short or too long due to manufacturing defect or environmental factors).

1. DEFINITION:

The digits that are considered correct and the first doubtful digit are called **significant figures**.

The number of significant figures in a measurement depends upon the precision of the instrument and, to some extent, upon the skill of the measurer. An effort should always be made to obtain as many figures as an instrument will allow. Conversely, only significant figures should be recorded in taking data – i.e., a measurement should not be written in such a way as to imply a greater precision than is actually inherent in the measuring device and/or measuring technique.

Example 1: We might use a variety of instruments to determine the mass of a typical calculator, and obtain the following results:

0.2 kg (1 significant figure) on a bathroom scale
0.185 kg (3 significant figures) on a good supermarket scale
0.183265 kg (6 significant figures) on an analytical balance.

In the above example, we could express the results in grams instead of kilograms – clearly, the precision of the result would be unaffected by a simple change of units, so that we would still end up with the same number of significant figures.

Example 2: $0.183265 \text{ kg} = 183.265 \text{ g}$
 $\phantom{0.183265 \text{ kg}} = 183265 \text{ mg}$
 $\phantom{0.183265 \text{ kg}} = 0.000183265 \text{ ton}$

All numbers have the same number of significant figures. This implies that the position of the decimal point has no effect on the number of significant figures. Zeros which are used merely to locate a decimal point are not significant figures but only “spacers”. On the other hand if a zero represents a value actually obtained from an instrument, then it is a significant figure.

Example 3: In the number 0.008040, the last two zeros are significant figures but the first three are not.

Zeros must be used even with whole numbers if the measurement is to convey a certain degree of precision.

Example 8:

Result of a calculation as read from a calculator display *	Number of significant figures	Result correctly rounded off	Comments
Fuel economy of $0.25\bar{3}68$ l/km	3	0.254 l/km	A real gas-guzzler!
Engine displacement of $38\bar{2}4$ cm ³	2	3.8×10^3 cm ³	One reason why it is a gas-guzzler!
Engine power output of $96.\bar{5}$ kW	2	97 kW or 96 kW	=130 hp for that gas-guzzler.

* The bar placed over a digit indicates the last significant figure (i.e., the first doubtful figure)

3.2 Addition and Subtraction

Arrange the quantities to be added or subtracted in column form and carry out the required arithmetical operation. Then scan the columns from left to right ...

RULE A: THE LAST SIGNIFICANT FIGURE IN THE RESULT IS THE FIGURE UNDER THE LAST COLUMN WHICH CONTAINS ONLY SIGNIFICANT FIGURES.

Example 9: Add the following measured lengths:

$80\bar{0}$ m	To obtain the 8 in the sum, an unknown figure was unjustifiably assumed to be zero (in the first line). Thus, the result must be rounded off to 946 m.
$136.\bar{5}$ m	
$9.\bar{3}$ m	
<hr/> 945.8 m	

3.3 Multiplication and Division

RULE B: THE NUMBER OF SIGNIFICANT FIGURES IN THE RESULT IS THE SAME AS THE NUMBER OF SIGNIFICANT FIGURES IN THE LEAST PRECISE QUANTITY INVOLVED IN THE CALCULATIONS.

Example 10: Calculate the volume of a cylinder whose height and diameter were found to be $20.3\bar{2}$ cm and $4.1\bar{0}$ cm, respectively

$$V = \frac{1}{4} \pi D^2 H = \frac{1}{4} \pi (4.1\bar{0} \text{ cm})^2 (20.3\bar{2} \text{ cm}) = 268.\bar{2}76 \text{ cm}^3$$

$$= 268 \text{ cm}^3$$

The fourth figure in $4.1\bar{0}$ is unknown. The value of this unknown figure would affect the values of the fourth figure in the result but would not significantly change the third figure, namely 8.

What about the significant figures in “ $1/4$ ” and “ π ”? (See Example 5 and the comment that precedes it).

Example 11:

Calculate the power required to raise a $92\bar{5}$ kg elevator $3\bar{5}$ m in $137.\bar{5}$ seconds at a constant speed, neglecting friction.

$$P = \frac{mg\Delta y}{\Delta t} = \frac{(92\bar{5} \text{ kg})(9.806 \text{ m/s}^2)(3\bar{5} \text{ m})}{137.\bar{5} \text{ s}} = 2\bar{3}09 \text{ J/s}$$

$$= 2.\bar{3}1 \text{ kW}$$

$$= 2.3 \text{ kW}$$

The least precise quantity ($3\bar{5}$ m) has only two significant figures, which limits the precision of the result to two significant figures.

3.4 Combined Operations

Follow the usual hierarchy of operations.

Whenever you carry out addition or subtraction, apply rule A.

Whenever you carry out multiplication or division, apply Rule B.

Example 12:

When the temperature of a solid bar is raised from T_1 to T_2 , the length of the bar changes from L_1 to L_2 according to

$$\Delta L = L_2 - L_1 = \alpha L_1 (T_2 - T_1)$$

where α is the coefficient of linear expansion of the material in question (average value for this temperature range).

Find L_2 using the following measured values:

$$\alpha = 1\bar{5} \times 10^{-6} / \text{C}^\circ, L_1 = 58.\bar{5} \text{ cm}, T_1 = 2\bar{4} \text{ }^\circ\text{C}, T_2 = 99.\bar{7} \text{ }^\circ\text{C}.$$

(Note: If T_2 is the actual boiling point of water, it can be determined more precisely than T_1 , using the known relationship between temperature and the vapor pressure of water.

$$L_2 = L_1 + \alpha L_1 (T_2 - T_1)$$

First operation
(apply rule A)

$$L_2 = 58.\bar{5} + 1\bar{5} \times 10^{-6} \times 58.\bar{5} \times (99.\bar{7} - 2\bar{4})$$

$$L_2 = 58.\bar{5} + 1\bar{5} \times 10^{-6} \times 58.\bar{5} \times (7\bar{5}.7)$$

Second and third
operations (apply rule B)

$$L_2 = 58.\bar{5} + 0.06\bar{6}43$$

$$\therefore L_2 = 58.\bar{5}664 \text{ cm} = 58.6 \text{ cm}$$

Fourth operation
(apply rule A)

3.5 General Notes

- In calculations with measured quantities it is best to work through the intermediate calculations with one figure more than the final result will contain in order to avoid the “accumulated round-off error”. The final result should be rounded off to the correct number of significant figures.
- The procedures for treating significant figures outlined herein do not cover all situations. Unique cases will have to be treated on their own merit.

Exercises on Significant Figures

(Pre-lab questions for Lab #1)

1. Write the following quantities in standard form, assuming three significant figures in each:

- 5360 W
- 20,000 V
- 9500.5 A
- 12345.6 m
- 0.0004177 kg
- 865.51 cm

In all the following exercises, only the significant figures have been kept in all the measured quantities.

2.

Add the following measured lengths:

45.7 cm; 2032 cm; 2.5688×10^2 m; 12.7×10^{-2} km; 1.00001×10^3 mm

3. A certain rod measures 1.0000 m in length at -10.0 °C and 1.0008 m at $+25.0$ °C. Use a formula from Example 12 to determine which of the following gives the correct values of α .

- a. $2.2857 \times 10^{-5}/\text{C}^\circ$ b. $2.29 \times 10^{-5}/\text{C}^\circ$ c. $2.3 \times 10^{-5}/\text{C}^\circ$ d. $2 \times 10^{-5}/\text{C}^\circ$

4. The time taken by sunlight to reach the Earth is given by:

$$t = \frac{\text{distance between Earth and Sun}}{\text{speed of light in vacuum}} = \frac{15.0 \times 10^7 \text{ km}}{(2.9979 \times 10^5 \text{ km/s})(60 \text{ s/min})}$$

A calculator gives 8.33917 (verify this). What is the correct answer (including units)?

5. Calculate the weight of 37 cars if each weighs 1.283×10^4 N.

6. Re-calculate the answer in Example 11, using

- a. $\Delta y = 36$ m b. $\Delta y = 34$ m

7. Calculate the surface area of a cylindrical tank whose radius and height were found to be 1.320 m and 0.445 m, respectively. Using the formula

$$A = 2(\pi R^2) + (2\pi R)H$$

the calculator yields $10.947822 \text{ m}^2 + 3.690743 \text{ m}^2 = 14.638565 \text{ m}^2$. What is the correct answer?

8. For a cart rolling down an inclined plane, the formula $\Delta s = v_0 t + \frac{1}{2} a t^2$

gives the displacement Δs as a function of time t . Given the following measurements: v_0 (initial velocity) = 0.12 m/s down the plane, $\Delta s = 1.325 \text{ m}$, $t = 1.906 \text{ s}$, find the acceleration a .